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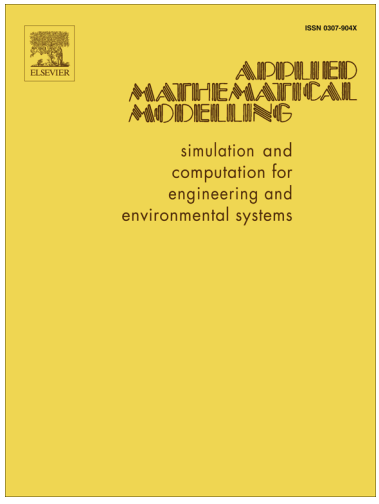

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Robust Tuning of Dynamic Matrix Controllers for First Order plus Dead Time Models

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Dynamic Matrix Control is a widely used Model Predictive Controller in industrial processes. The successful implementation of dynamic matrix control in practical applications requires appropriate tuning of the controller parameters. Tree different cases are considered. In the first case, a tuning formula is developed that ensures the nominal closed loop desired performance. However, this formula may fail in the presence of plant uncertainty. Therefore a lower bound for the tuning parameter is derived to secure the robust stability of the uncertain first order plus dead time plant. Finally, a tuning boundary is derived which gives the lower and upper permissible bounds for the tuning parameter that guarantee the robust performance of the uncertain first order plus dead time plant. The tuning procedure is based on the application of Analysis of Variance, curve fitting and nonlinear regression analysis. The derived results are validated via simulation studies and some experimental results.

Key words: Dynamic matrix control; Robust tuning; First order plus dead time models; Analysis of variance; Curve fitting.

1. Introduction

Model Predictive Control (MPC) strategies have been successfully employed in a wide range of real industrial applications [1-3]. It is shown in [4], that linear MPC as a modern advanced control methodology improves both the energy conservation and productive capacity in industrial and chemical processes. Dynamic Matrix Control (DMC) as one of the first commercial implementations of MPC is extensively used in many chemical and petrochemical processes [5]. This is due to the simple structure of the controller and the minimum required plant information. Many open loop stable processes can be effectively modeled by Finite Step Response (FSR) models [5]. DMC uses the FSR information which is easily obtained by open loop step tests. Two key DMC elements are a FRS based prediction model and an objective function [6]. It is desired that the future plant output on a specified finite horizon follow a desired reference trajectory and simultaneously the control effort is minimized.

DMC has many tuning parameters such as the prediction horizon, the control horizon, the model horizon, and the penalty weights in the objective function, the reference trajectory time constant and the sampling time. Note that the successful implementation of DMC in practical applications requires a proper tuning of the controller parameters. The issue of MPC tuning is addressed in many research papers [7, 8].

In Lee and Yu [9], a tuning method to obtain robust performance is proposed based on state estimation and sensitivity functions analysis. A case dependent method for MPC tuning, called the response surface tuning is proposed for a pressure tank system in [10]. An on-line tuning strategy is presented in [11] for DMC, based on a constrained least square optimization that tunes parameters to satisfy a predefined closed loop time domain performance. Trierweiler and Farina [12] used the Robust Performance Number (RPN) to develop a MPC tuning procedure for multivariable and non minimum phase plants. An extension of the modified GPC algorithm and a tuning strategy is developed in [13]. Garriga and Soroush [14] used MATHEMATICA to present an analytical study of the effects of various MPC tunable parameters on the location of closed-loop eigenvalues. This idea leads to complicated formulations not straightforwardly applicable to tune the parameters for desired performance. Two methods for selecting the MPC weight matrices are derived by Cairano and Bemporad [15] that use a state space MPC approach, multivariable controllers are also considered and Linear Matrix Inequality (LMI) solves the inverse problem of controller matching which numerically tunes the weight matrices. Shah and Engell [16] provided a tuning procedure to achieve desired closed loop performance. The penalty weights and control horizon of GPC are tuned by using convex optimization. However, it is noted that the control horizon must be one; otherwise the optimization problem would no longer be solved using the convex optimization. Recently, a robust analysis and tuning for GPC in two-degree-of-freedom

configuration is proposed in [17]. Note that, none of the above mentioned papers provide a tuning formula. Shridhar and Cooper [18] derived a closed form equation for all the DMC parameters based on the FOPDT model approximation of the real plant. Among these tuning parameters, the move suppression coefficient, λ (defined later) is the most effective parameter [18] and the equation given for λ is to avoid singularity calculation in the control signal and closed loop performance is not considered. An analytical formulation for DMC tuning using gain and dead time of the plant based on some practical approaches is presented in [19], but the weight factor of control effort is not normalized that can lead to difficulties as shown in [18]. Analysis of Variance (ANOVA) is used in [20] to develop an analytical equation for λ. Unfortunately, there are severe deficiencies associated with the derived formulae [21, 22]. The ANOVA technique is used for tuning GPC parameters in the case of Second Order plus Dead Time (SOPDT) models in [23], and a new analytical equation for λ is obtained. An analytical tuning equation for DMC tuning parameter λ is developed by Bagheri and Khaki-Sedigh [21, 22]. It is based on the application of ANOVA and nonlinear regression analysis for FOPDT process models and provides closed form tuning equations for the nominal performance and disturbance rejection of DMC. Recently, Bagheri and Sedigh [24] developed an analytical MPC tuning methodology for FOPDT models to ensure nominal desired performance of closed loop system. Also, achievable performance is addressed. This tuning strategy is extended to unstable plants with

fractional dead times in [25] and also for multivariable plants in [26]. Note that, in Bagheri and Khaki-Sedigh [21, 22, and 24-26] only nominal FOPDT model is considered to achieve tuning equations. In this paper, the idea in Bagheri and Sedigh [21, 22] is extended to handle the uncertain FOPDT systems. A tuning equation and inequalities are derived for DMC. The focus is on tuning the parameter λ, and three formulations are proposed. First, the nominal model without uncertainty is considered and the objective would be the desired nominal closed loop performance. This performance is the ratio of the closed loop to the open loop settling times. Then, robust stability is the tuning objective for an uncertain FOPDT model with structured uncertainty which gives a lower band of the tuning parameter λ. Finally, the robust performance of an uncertain FOPDT model is considered that results in a lower and upper bound for λ. In the first stage, a bank of nominal FOPDT models with different performance parameters is defined. In the case of uncertainty, a bank of uncertainties is defined and simulated to test the effect of different model parameters, uncertainties and stability/performance parameter on the tuning parameter λ. In the second stage, ANOVA is performed on these data to determine the effectiveness of the parameters on the tuning parameter. Note that, there are several parameters that affect the tuning parameter λ and ANOVA is an appropriate tool to provide information about the relative effect of the various parameters on the tuning parameter. Finally, curve fitting

and nonlinear regression techniques are employed to obtain a closed form tuning equation for λ .

In the following section, the preliminary materials on DMC and the previous DMC tuning methods that lead to closed form formulations are briefly studied. In section 3, the procedure for the proposed tuning method is given. Finally, experimental results are given in section 4.

2. Preliminary Materials

2.1 Dynamic Matrix Control (DMC)

DMC is a widely used industrial MPC method which was developed in the early 1970s and is a practical control system design, particularly in the oil and petrochemical industries [2]. In this section, Single Input-Single Output (SISO) formulation of the DMC is briefly reviewed.

In the open loop stable plants with slow and simple dynamics such as the petrochemical processes, FSR models can sufficiently capture the process dynamics [5], which are readily employed by DMC. Assume that $y(t)$ is the process output, $u(t)$ is the control signal and $\Delta u(t) = u(t) - u(t-1)$ is the control effort. Then, the system step response can be described as follows

$$y(t) = \sum_{i=1}^{\infty} g_i \Delta u(t-i) \tag{1}$$

where g_i are the sampled step responses. The output prediction values along the finite horizon will be as

$$\hat{y}(t+k\,|\,t)=\sum_{i=1}^{k} g_i \Delta u(t+k-i)+f(t+k) \tag{2}$$

where $f(t+k)$ is the free response of the system, and is given by

$$f(t+k)=\sum_{i=1}^{N}(g_{k+i}-g_i)\Delta u(t-i)+y_m(t) \tag{3}$$

and $y_m(t)$ is the measured output value. We have $g_{k+i}-g_i \approx 0,\ i>N$ where, N is the model horizon. In the vector form we have

$$\hat{\boldsymbol{y}}=\boldsymbol{G}\boldsymbol{u}+\boldsymbol{f} \tag{4a}$$

where

$$\hat{\boldsymbol{y}} = \begin{bmatrix} \hat{y}(t+1|t) \\ \hat{y}(t+2|t) \\ \vdots \\ \hat{y}(t+P|t) \end{bmatrix}, \quad \boldsymbol{u} = \begin{bmatrix} \Delta u(t) \\ \Delta u(t+1) \\ \vdots \\ \Delta u(t+M-1) \end{bmatrix}, \quad \boldsymbol{f} = \begin{bmatrix} f(t+1) \\ f(t+2) \\ \vdots \\ f(t+P) \end{bmatrix}$$

$$\boldsymbol{G} = \begin{bmatrix} g_1 & 0 & \cdots & 0 \\ g_2 & g_1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ g_M & g_{M-1} & \ddots & g_1 \\ \vdots & \vdots & \ddots & \vdots \\ g_P & g_{P-1} & \cdots & g_{P-M+1} \end{bmatrix} \tag{4b}$$

and P is the output horizon, M is the control horizon and $\boldsymbol{G}$ is the dynamic matrix. The quadratic objective index in the DMC structure is

$$J = \sum_{j=1}^{P} \left[\hat{y}(t+j) - w(t+j)\right]^2 + \sum_{j=1}^{M} \lambda \left[\Delta u(t+j-1)\right]^2 \tag{5}$$

where w is the desired reference trajectory and λ is the move suppression coefficient, which is an important tuning parameter in DMC. The control signal is calculated as follows

$$\boldsymbol{u} = (\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G} + \lambda \mathrm{I})^{-1}\boldsymbol{G}^{\mathrm{T}}(\boldsymbol{w} - \boldsymbol{f}), \quad \boldsymbol{w} = \begin{bmatrix} w(t+1) \\ w(t+2) \\ \vdots \\ w(t+P) \end{bmatrix} \tag{6}$$

Consequently, the DMC tuning parameters can be listed as λ, P, N, M and T_s. Note that T_s is the sampling time. Among these tuning parameters, λ is the most effective [18] and in this paper a tuning equation is given for these parameters, but the focus is on λ.

2.2 DMC Tuning Methods

There are a number of DMC tuning strategies in the literatures, but only a few provide analytical and closed form expressions for the tuning parameters which are briefly introduced here. A pioneer paper in this field is developed by Shridhar and Cooper [18] that provides a tuning formulation for the move suppression coefficient λ based on the following FOPDT model

$$\mathrm{G}_m(s) = \frac{ke^{-\theta s}}{\tau s + 1} \tag{7}$$

However, the derived formula neglects the closed loop performance considerations. Wojsznis et al. [19] addressed some practical guidelines for MPC tuning and derived an analytical formulation for λ in the multivariable case. For SISO plants the formula is

$$\lambda = 9\left(1 + \frac{6\theta}{P} + \frac{3k\theta}{P}\right)^2$$

It is shown in (Shridhar and Cooper 1997) that λ can be scaled according to the plant

dc gain, i.e. $\lambda = fk^2$. So the above formulation in dealing with large value of dc gains leads to a fast response and also small value of dc gain leads to a slow response.

The ANOVA technique is used by Iglesias et al. [20] to obtain an analytical equation for λ. But there are several deficiencies associated with the derived formulae as stated in [21,22]. Bagheri and Khaki-Sedigh [21, 22] developed a closed form tuning equation for λ. It is based on the application of ANOVA and nonlinear regression analysis for FOPDT process models. The performance index that is used to obtain the tuning equation is

$$J = \int_0^{t_f} \left(\left(y(t) - w(t) \right)^2 + \Gamma \left(\Delta u(t) \right)^2 \right) dt \tag{8}$$

where Γ is a user specified weighting parameter and t_f is the final simulation time. Note that both tracking and disturbance rejection is considered. The effectiveness of the tuning formulation is shown by simulation and experimental tests [21, 22]. To summarize, the above tuning equations are presented in table 1.

3. The Tuning Procedure

In this section, we first deal with the tuning of T_s, P, N and M. Then, the parameter λ will be considered. To achieve this, a desired closed loop nominal performance is determined and a procedure is presented to obtain a formula for the DMC tuning parameter λ to ensure the desired closed loop nominal performance. To evaluate the

efficiency of the derived formula, simulation results are presented at the nominal operating point. However, uncertainties are inevitable in real industrial plants and therefore a robustness analysis is performed. First, robust stability is considered and a lower bound for λ to ensure closed loop stability in the presence of structured model uncertainties is derived. Then, robust performance is depicted and a tuning region for λ to ensure closed loop robust performance is derived. Finally, a pH neutralization process is used to demonstrate the effectiveness of the proposed methodology.

The sampling time selection shown in table 1 can produce erroneous results in some cases. Consider a plant with large delay time and a fast dynamic, which gives a small value of τ. Table 1 could propose a large sample time that is not correct. Hence, the sampling time selection is suggested as

$$T_s \approx \frac{t_s}{40} \qquad t_s \approx 4\tau + \theta \quad (2\% \text{ criterion}) \tag{9}$$

where t_s is the open loop settling time. A settling time to within 2% of the system is considered. So we have

$$T_s \approx \frac{4\tau + \theta}{40} \tag{10}$$

It is well known that P should be chosen to include all the transient behaviors, and

therefore it heuristically selected as

$$P = 40 \tag{11}$$

As shown in [21, 22], N should be larger than P. Let

$$N = 80 \tag{12}$$

Note that, $y(NT_s) \approx 0.9997 y_{ss}$ where y_{ss} is the steady state value of $y(t)$. Finally, the control horizon is chosen such that a tradeoff between costing and performance is included, so

$$M = 4 \tag{13}$$

3.1 The Nominal Performance

The key tuning parameter is λ and in this section, a tuning formula for λ is derived to give the desired nominal closed loop performance. Consider the FOPDT plant given by (7). Then, the settling time with a 5% criterion

$$t_s \approx 3\tau + \theta \tag{14}$$

The equivalent closed loop transfer function is approximately

$$G_c(s) \approx \frac{e^{-\theta_c s}}{\tau_c s + 1} \tag{15}$$

where $\theta_c = \theta + T_s$. The following performance index is introduced as

$$\frac{\tau}{\tau_c} \approx \frac{t_s - \theta}{t_{cs} - \theta_c} \tag{16}$$

where t_{cs} is the closed loop settling time. Hence, the time domain criteria used in this paper is defined as

$$r = \frac{t_s}{t_{cs}} \quad \text{(5\% criterion)} \tag{17}$$

This criterion shows the ratio of the closed loop to the open loop settling times which is indicative of the change in closed loop response speed. Note that, by choosing $r > 1$, the closed loop response gets faster and by $r < 1$ the closed loop response becomes slower in comparison with the open loop response.

Dealing with the FOPDT model, the values of τ and θ are not individually important parameters, but θ / τ is an important parameter. So in the following θ / τ is proposed to shape the performance. Also, it is shown by Shridhar and Cooper [18] that $\lambda = fk^2$. This choice scales plant according to the dc gain of the FOPDT model. Therefore, in the following the goal is to find an optimal equation for f and we choose $k = 1$ for simulations.

The tuning formulation is now summarized in the following steps:

(1) Define the sets

$$\mathrm{A} = \left\{ \frac{\theta}{\tau} : a_{min} \le \frac{\theta}{\tau} \le a_{max} \right\}$$
$$\mathrm{R} = \{ r : r_{min} \le r \le r_{max} \}$$
$$\mathrm{F} = \{ f : f_{min} \le f \le f_{min} \}$$

These sets are discrete and the step values are chosen based on a trade-off between accuracy and computation cost;

(2) Let $k = 1$;

(3) Chose a value for θ / τ from set A, consider $\tau = 1$ and find θ;

(4) Compute the settling time for selected τ and θ from the open loop FOPDT model according to (14) and subtract the time delay θ from the computed time value;

(5) Choose a value for r from set R;

(6) Divide the time value computed in step (4) by selecting r in step (5) and add the time delay θ to it. The result is the desired settling time of the closed loop system;

(7) Simulate the closed loop system for the largest f belonging to F with respect to the selected τ and θ. Compute the settling time of the simulated system. If the computed value is smaller than the desired value that is obtained in step (6) then f is selected as the response. Otherwise the next f of F which is smaller

than the prior one is selected and this step is repeated until the desired f is achieved. So by finding f the response set $(\theta/\tau, r, f)$ created;

(8) Return to Step (5) and repeat steps (5) to (7) for the next r and compute the all response sets;

(9) Return to step (3) and repeat steps (3) to (7) for the next θ/τ from A set. So a bank of responses sets in the $(\theta/\tau, r, f)$ format is achieved;

(10) Check the effect of r and θ/τ on f by the ANOVA method;

(11) Provide a structure for tuning formula of f by depicting f curves with respect to r and θ/τ and considering the computed effect values in step (10);

(12) Finally, compute the optimum parameters of the tuning formula's structure of step (11) by nonlinear fitting and curve fitting tools of MATLAB.

The above procedure can easily be implemented. By following step (1) to (9) optimal values for the parameter f are achieved. For the ANOVA test in step (10), one way combination between the parameters is taken into account because more than one combination leads to infinity in the F test.

Application of the tuning procedure. The above procedure is now implemented for the following parameters

$$a_{min} = 0,\ a_{max} = 1.5,\ r_{min} = 0.5,\ r_{max} = 4,\ f_{min} = 0,\ f_{max} = 500$$

Results of the ANOVA are presented in table 2. According to F-values and P-values, performance parameter, r, is more effective than θ/τ.

In step (11), a structure should be chosen. In Figure 1, the behavior of optimal f versus r in three different cases for θ/τ is shown. Using curve fitting techniques, a rational structure is chosen as

$$f = \frac{a_1 r^3 + a_2 r^2 + a_3 r + a_4}{r^3 + b_1 r^2 + b_2 r + b_3} \tag{18}$$

By trial and error choosing a_1 and b_3 a function of θ/τ, similar curves are obtained. Hence,

$$a_1 = c_1 + c_2\left(\frac{\theta}{\tau}\right),\ b_3 = d_1 + d_2\left(\frac{\theta}{\tau}\right) \tag{19}$$

Finally, in step (12) optimum parameter values are obtained. This procedure yields the following tuning formula

$$\lambda = fk^2,\ f = \frac{\left(-2.333 - 0.166\left(\frac{\theta}{\tau}\right)\right) r^3 + 18.24 r^2 - 38.02 r + 28.3}{r^3 - 1.968 r^2 + 1.338 r + \left(-0.277 - 0.024\left(\frac{\theta}{\tau}\right)\right)} \tag{20}$$

To show the effectiveness of this formula, consider the following system

$$G_m(s) = \frac{3e^{-10s}}{20s+1}$$

Let the desired closed loop response to be 3 times faster than the open loop response, that is $r = 3$. According to (28) we obtain $\lambda = 9.1$. Figure 2 shows the closed loop responses. It is shown that the 5% settling time criterion is achieved with excellent accuracy.

Now let it be desired to have closed loop response 0.8 times that of the open loop response. According to (20) we obtain $\lambda = 1294$. Figure 3 shows the closed loop responses for this case.

To test the validity of the derived formula, 20 different cases are randomly selected as FOPDT models in the interval of [0 1.5] and the desired rates in the interval of [0.5 4] are chosen as bellow

$$\frac{\theta}{\tau} = 0.2,\ 0.4,\ 0.9,\ 1.4$$
$$r = 0.6,\ 1,\ 1.7,\ 2.7,\ 3.2$$

For each model and each desired rate, the index error $(|r - r_d|/r_d)\times 100\%$ is calculated, where r_d is the desired rate and r is the rate of response. By averaging this index for all tests 2.7% error is obtained. The worth case with 9.4% error corresponds to $\theta/\tau = 1.4,\ r = 3.2$ and the best one is with 0.45% error corresponds to

$\theta / \tau = 0.9,\ r = 2.7$.

3.2 Robust Stability

In this section, a lower bound for tuning parameter λ to ensure robust stability is derived. Let the real plant be modeled by a FOPDT transfer function. It is assumed that the plant belongs to the following structured set G

$$G = \left\{ \tilde{G} = \frac{\tilde{k} e^{-\tilde{\theta} s}}{\tilde{\tau} s + 1} \;\middle|\; k_{min} \le \tilde{k} \le k_{max},\ \tau_{min} \le \tilde{\tau} \le \tau_{max},\ \theta_{min} \le \tilde{\theta} \le \theta_{max} \right\} \tag{21}$$

This can be parameterized by a finite number of parameters $(\tilde{k}, \tilde{\tau}, \tilde{\theta})$. The uncertain plant transfer functions can be equivalently written in the following multiplicative uncertainty form

$$\tilde{G} = \frac{\tilde{k} e^{-\tilde{\theta} s}}{\tilde{\tau} s + 1} = G_m (1 + \Delta_W) \tag{22}$$

Note that G_m is the nominal model of the uncertain plant and Δ_W is the model uncertainty. It can be shown that the uncertain parameters can be rewritten as

$$\tilde{k} = k(1 + \Delta_k),\quad \tilde{\tau} = \tau(1 + \Delta_\tau),\quad \tilde{\theta} = \theta(1 + \Delta_\theta) \tag{23a}$$

where

$$|\Delta_k| \le \frac{k_{max}}{k} - 1, \quad |\Delta_\tau| \le \frac{\tau_{max}}{\tau} - 1, \quad |\Delta_\theta| \le \frac{\theta_{max}}{\theta} - 1 \tag{23b}$$

And the nominal parameters are chosen as

$$k = \frac{k_{min} + k_{max}}{2}, \quad \tau = \frac{\tau_{min} + \tau_{max}}{2}, \quad \theta = \frac{\theta_{min} + \theta_{max}}{2} \tag{23c}$$

Hence,

$$\Delta_W(s) = \frac{\tilde{G}}{G_m} - 1 = \frac{k(1+\Delta_k)e^{-\theta(1+\Delta_\theta)s}}{\tau(1+\Delta_\tau)s+1} \frac{\tau s+1}{ke^{-\theta s}} - 1 = (1+\Delta_k)\frac{\tau s+1}{\tau(1+\Delta_\tau)s+1} e^{-\theta\Delta_\theta s} - 1 \tag{24}$$

Application of the small-gain theorem gives the sufficient condition for robust stability as

$$\| \Delta_W \mathrm{T}(j\omega) \|_\infty < 1 \tag{25}$$

where $\mathrm{T}(j\omega)$ is the complementary sensitivity function of the closed loop system. The following steps produce the tuning formula for robust stability:

(1) First define the sets

$$\mathrm{A} = \left\{ \frac{\theta}{\tau} : a_{min} \le \frac{\theta}{\tau} \le a_{max} \right\}$$
$$\mathrm{D} = \{ (\Delta_k, \Delta_\tau, \Delta_\theta) \mid \Delta_{k\,min} \le \Delta_k \le \Delta_{k\,max}, \Delta_{\tau\,min} \le \Delta_\tau \le \Delta_{\tau\,max},\ \Delta_{\theta\,min} \le \Delta_\theta \le \Delta_{\theta\,max} \}$$
$$\mathrm{F} = \{ f : f_{min} \le f \le f_{min} \}$$
$$\mathrm{W} = \{ \omega : \omega_{min} \le \omega \le \omega_{max} \}$$

These sets are discrete and the step values are chosen based on a trade-off between the desired accuracy and computation cost;

(2) Let $k = 1$;

(3) Choose a value for θ / τ from the set A, consider $\tau = 1$ and find θ;

(4) For the selected τ and θ, chose Δ_k, Δ_τ and Δ_θ from the set D. Then for the set $(\theta / \tau, \Delta_k, \Delta_\tau, \Delta_\theta)$ according to (33) find $\Delta_W(s)$;

(5) Chose a value for ω from the set W. Let $r(t) = \sin(\omega t)$ with a chosen frequency;

(6) Simulate the closed loop system for the smallest f that belongs to F;

(7) According to the maximum steady state output value, find $|\mathrm{T}(j\omega)|$ For a chosen ω find $|\Delta_W(j\omega)|$ according to $\Delta_W(s)$ obtained in step (4);

(8) Check the sufficient condition for robust stability (25), if it is satisfied, go to step (5) and repeat step (5) to step (8) for the next frequency value. If for all of the frequencies of the set W, the condition is satisfied then f is selected as the response. In the case that condition is not satisfied, go to step (6) and select the

next f of F which is larger than the prior one. These steps are repeated until the desired f is achieved. So by computing f the response set $(\theta/\tau, \Delta_k, \Delta_\tau, \Delta_\theta, f)$ is determined;

(9) Return to step (4) and repeat steps (4) to (8) for the next $(\Delta_k, \Delta_\tau, \Delta_\theta)$ and compute the response sets;

(10) Go to step (3) and repeat steps (3) to (9) for the next θ/τ from A. So a bank of response sets in the $(\theta/\tau, \Delta_k, \Delta_\tau, \Delta_\theta, f)$ format is achieved;

(11) Check the effect of $\Delta_k, \Delta_\tau, \Delta_\theta$ and θ/τ on f by the ANOVA method;

(12) Provide a structure for the tuning formula of f by depicting f curves versus $\Delta_k, \Delta_\tau, \Delta_\theta$ and θ/τ and considering the computed effect value in step (11);

(13) Finally, compute the optimum parameters of the tuning structure of step (12) by nonlinear fitting and curve fitting techniques.

Application of the tuning procedure. The above procedure is now implemented for the following parameters

$$r_{min} = 0.5,\ r_{max} = 4,\ f_{min} = 0,\ f_{max} = 500,\ \omega_{min} = 0,\ \omega_{max} = 2\pi / T_s$$
$$\Delta_{k\,min} = 0,\ \Delta_{k\,max} = 1.5,\ \Delta_{\tau\,min} = 0,\ \Delta_{\tau\,max} = 1.5,\ \Delta_{\theta\,min} = 0,\ \Delta_{\theta\,max} = 1.5$$

Performing this procedure yields the following lower bound for λ to have robust stability

$$\lambda > k^2 \left(\frac{\left(0.2+0.75\frac{\theta}{\tau}\right)\Delta_\tau^{\,2} - 0.1\Delta_\tau + 0.03}{\Delta_\tau^{\,2} - 1.85\Delta_\tau + 1} + \frac{0.72\left(\frac{\theta}{\tau}\Delta_\theta\right)^{\frac{3.15\Delta_\theta^{\,2} - 2.15\Delta_\theta + 0.68}{\Delta_\theta^{\,2} - 0.5\Delta_\theta + 0.15}}}{\Delta_\theta^{\,2} + 0.01} + 0.001 \right) e^{\left(2.5+4\frac{\theta}{\tau}\right)\Delta_k^{\,1.3}} \quad (26)$$

In order to evaluate the robust stability tuning formula, an example is presented.

Consider the system with the following transfer function.

$$G_m(s) = \frac{e^{-8s}}{10s+1}$$

Consider that uncertainties are as follows

$$\Delta_k = 0.4,\ \Delta_\tau = 0.3,\ \Delta_\theta = 0.5$$

Now we choose the real plant transfer function as

$$G_p(s) = \frac{1.4e^{-11.2s}}{8s+1}$$

The tuning formulation proposed in [18] gives $\lambda = 0.242$.The derived bound for robust stability leads to $\lambda > 2.14$. So we chose $\lambda = 2.2$. Figure 4 shows the results of simulation. The stability of the response according to the proposed formulation is shown.

3.3 Robust Performance

Finally, in this section a tuning region for robust performance in the presence of parameterized structured uncertainties is developed. In section 3-1, a criterion for nominal performance is given. However, the desired performance should be an interval to handle the structured uncertainties. That is, the user specifies the lower bound and the upper bound of the desired r. To produce the tuning formula for desired robust performance, a similar procedure as in section 3-1 is adopted, but uncertainties are also inserted for the upper and lower bounds of r (r_H, r_L). This procedure yields to the following boundary

$$k^2 \mathrm{F}_1\left(r_H, \frac{\theta}{\tau}\right)\left(f_{H1} f_{H2} f_{H3}\right) < \lambda < \frac{k^2 \mathrm{F}_1\left(r_L, \frac{\theta}{\tau}\right)}{\left(f_{L1} f_{L2} f_{L3}\right)} \tag{27}$$

where

$$\mathrm{F}_1\left(r,\frac{\theta}{\tau}\right)=\frac{\left(-2.333-0.166\left(\frac{\theta}{\tau}\right)\right)r^3+18.24r^2-38.02r+28.3}{r^3-1.968r^2+1.338r+\left(-0.277-0.024\left(\frac{\theta}{\tau}\right)\right)}$$

$$f_{H1}=e^{\left(0.5+6\left(\frac{\theta}{\tau}\right)^{1.3}+\frac{13.2}{r_H^2}\right)\Delta_k} \quad , \quad f_{L1}=e^{\left(23-\frac{7}{r_L^2}+5r_L^3\left(\frac{\theta}{\tau}\right)^{1.5}\right)\Delta_k^{0.9}} \tag{28}$$

$$f_{H2}=e^{\left(0.2+0.6\left(\frac{\theta}{\tau}\right)^{2}+0.06r_H^3\right)\Delta_\tau^{0.8}} \quad , \quad f_{L2}=e^{(0.1+0.53r_L)\Delta_\tau^{\frac{0.7-3\Delta_\tau}{r_L}}}$$

$$f_{H3}=e^{\left(1-0.015r_H^2\right)\Delta_\theta^{\frac{1}{1.2+\frac{\theta}{\tau}}}} \quad , \quad f_{L3}=e^{\left(0.2+2r_L^{2.5}\right)\Delta_\theta^{\frac{1}{0.8+1.5\left(\frac{\theta}{\tau}\right)^2}}}$$

To evaluate the robust performance tuning region, a simulation test on the pH neutralization process is presented. This process is a well-known and standard bench mark system for both single loop and also multivariable control strategies. Here we deal with the single input-single output pH process. A schematic diagram of the pH neutralization process is depicted in Figure **Error! Reference source not found.**5. It is shown that, this process consists of acid (HNO3), base (NaOH) and buffer (NaHCO3, NaOH) streams that are mixed in a vessel [27]. The acid, base and buffer flow rates are presented respectively by F_a, F_b and F_{bf}. Note that, in single input-single output case the acid flow rate is assumed to be fixed and the control signal is base flow rate F_b. The control objective is to control the value of the pH of the outlet stream. It is assumed that the level of the solution is fixed. It is proposed that the pH of the outlet stream is measured at a distance from the plant, which introduces a measurement time delay θ.

The dynamic equations for reaction invariants of the effluent solution (w_a, w_b) are given by

$$\frac{dw_a}{dt} = \frac{1}{V}(F_a(w_{aa} - w_a) + F_b(w_{ba} - w_a) + F_{bf}(w_{bfa} - w_a))$$
$$\frac{dw_b}{dt} = \frac{1}{V}(F_a(w_{ab} - w_b) + F_b(w_{bb} - w_b) + F_{bf}(w_{bfb} - w_b))$$

The static behavior of this process is given by

$$[\mathrm{H}^+]^4 + [\mathrm{H}^+]^3(K_{a1} - w_a) + [\mathrm{H}^+]^2(K_{a1}(K_{a2} - w_a - w_b))$$
$$+ [\mathrm{H}^+](-K_{a1}(K_w + K_{a2}(w_a + 2w_b))) - (K_w K_{a1} K_{a2}) = 0$$
$$\mathrm{pH} = -\log_{10}[\mathrm{H}^+]$$

For more details about the nominal pH parameters and operating conditions see Henson and Seborg [27]. This process is nonlinear. However, it can be described by a linear FOPDT model rather accurately at different operating points. The operating points in the following simulations ranges from pH=5.75 to pH=7. In this case, for $F_{bf} = 0.55\,\mathrm{ml\,s^{-1}}$ we have the nominal model as

$$G_m(s) = \frac{\mathrm{pH}(s)}{F_b(s)} = \frac{0.4e^{-30s}}{82s+1}$$

In this range of operating, the plant uncertainties are

$$\Delta_k = 0.22,\ \Delta_\tau = 0.09,\ \Delta_\theta = 0$$

Consider that, the control objective is to have a closed loop response between 0.8 up to 1.5 times faster than nominal open loop response (i.e. $0.8 < r < 1.5$). According to (27) we obtain

$$6.143 < \lambda < 9.17$$

So an appropriate choice can be $\lambda = 7.65$. For tracking we choose three different pH values with minimum, maximum and an average dc gain value. Figure 6 shows the results, it is shown that the desired performance is achieved with adequate accuracy. This figure shows that the response is near the desired boundary. Now consider the closed loop responses between 0.9 up to 1.3 times faster than the nominal open loop response. According to (27) we obtain

$$64.8 < \lambda < 19$$

So there is no solution for this case. A key point is that for large uncertainties, the desired response must be relaxed.

3.4 The Tuning Algorithm

In this section, the applications of proposed tuning algorithms are presented. The procedure is as follows:

Step 1: Choose a proper nominal FOPDT model for plant,

Step 2: Calculate tuning parameters due to (10), (11), (12) and (13),

Step 3: If the process can be described with the chosen FOPDT model with enough accuracy, go to step 4, else go to step 5,

Step 4: Choose r properly due to desired closed loop performance. Then tuning parameter λ is given by (20), the procedure is finished.

Step 5: Find maximum uncertainty bounds of gain, constant time and time delay, i.e. Δ_k, Δ_τ and Δ_θ,

Step 6: If robust stability is considered, then use (26), and if robust performance is preferred then choose r_H and r_L properly due to desired closed loop performance. Finally bounds of tuning parameter λ is given by (27), the procedure is finished.

4. Experimental Results

In this section, the proposed tuning equations are experimentally verified on lab-scaled level and pressure control systems.

4.1 The level control system

The schematic diagram of the plant and the actual laboratory scale level control systems are shown in Figures 7(a) and 7(b), respectively. The control objective is the water level control in tank 1. Outlet cock 8 determines the flow of the outlet stream from the bottom

of the main tank. A big reservoir tank 2 gathers outlet water and a pump 3 circulates the water. Flow of the pumped water is controlled by a control valve 5. This water pours to the main tank. A Level sensor measures the water level in the main tank. Controller 6 should observe this measurement and apply appropriate command to the control valve. The control objective in this process is the water level control. The plant output range is 0 to 60 centimetres. Generally, the behaviour of plant is nonlinear, but it can be described by a FOPDT model with sufficient accuracy in each operating area. The dynamic of this system changes by changing the outlet cock. Hence, in two different situations we examine our tuning formulations. First, consider the output varies from 10cm to 40cm and the outlet cock is set to 5. This range of operation is then divided to smaller operating areas and for each one a FOPDT model is derived. Hence, the following uncertain model is derived

$$G_m(s) = \frac{5.85e^{-7s}}{27s+1}$$
$$\Delta_k = 0.1,\ \Delta_\tau = 0.15,\ \Delta_\theta = 0.13$$

Now consider that we want to have closed loop responses satisfying

$$0.9 \le r \le 1.7$$

According to (27) we obtain

$$9.87 \le f \le 36.28 \rightarrow 338 \le \lambda \le 1242$$

let

$$f = 15 \rightarrow \lambda = 515$$

so, the DMC tuning parameters are as follows

$$T_s = 3,\ P = 40,\ N = 80,\ M = 4,\ \lambda = 515$$

The experimental results are shown in Figure 8. It is shown that in all of the tracking set points the desired performance is achieved perfectly.

In the second test, the range of 10cm to 50cm is chosen and the outlet cock is set on value 15, hence we have

$$G_m(s) = \frac{6.5e^{-8s}}{30s+1}$$
$$\Delta_k = 0.06,\ \Delta_\tau = 0.2,\ \Delta_\theta = 0.13$$

Now consider that we want to have closed loop responses satisfying

$$1.1 \le r \le 1.9$$

According to (27) we obtain

$$5.5 \le f \le 8.2 \rightarrow 232.4 \le \lambda \le 346.5$$

let

$$f = 7 \rightarrow \lambda = 296$$

The experimental results are shown in Figure 9. It is shown that the desired set point tracking performance is achieved perfectly.

4.2 The pressure control system

The schematic diagram and the actual lab-scale pressure plant are shown in Figures 10(a) and 11(b), respectively. This plant includes two pressure tanks and the control objective is pressure control in these tanks. The output variation range is 0 to 220 and the selected range is 120 to 180. The behaviour of plant in this operating area is nonlinear. We divide the operation area in to the small enough operating areas such that in each one, a FOPDT model covers the behaviour of plant. So the following uncertain model is derived

$$G_m(s) = \frac{5.9e^{-14s}}{135s+1}$$
$$\Delta_k = 0.13,\ \Delta_\tau = 0.07,\ \Delta_\theta = 0.08$$

Now consider that we want to have closed loop responses satisfying

$$1 \le r \le 2$$

According to (27) we obtain

$$5.25 \le f \le 24 \rightarrow 182.7 \le \lambda \le 835.4$$

let

$$f = 20 \rightarrow \lambda = 696$$

So, the DMC tuning parameters are as follows

$$T_s = 14,\ P = 40,\ N = 80,\ M = 4,\ \lambda = 696$$

The experimental results are shown in Figure 11. It is shown that in all of the tracking set points the desired performance is achieved.

5. Conclusions

Tuning relationships are derived for single input-single output DMC. For all of the DMC parameters a tuning formula is derived but the focus in this paper is on λ. This parameter determines the quality of the closed loop responses. Using a FOPDT model, three analytically function of FOPDT parameters and desired performance parameters are obtained via ANOVA, data fitting and nonlinear regression. This leads to a closed form formula for tuning the DMC parameter in the case of nominal performance. For the robust stability of the uncertain FOPDT plant, a lower bound on λ is given and for its robust performance uncertainty tuning region is derived. Simulation and experimental studies are used to show the effectiveness of the proposed method.

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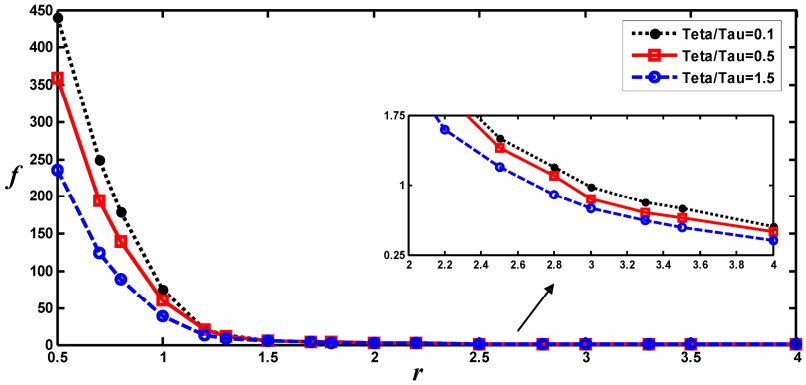

Figure 1. Optimal f via other parameters.

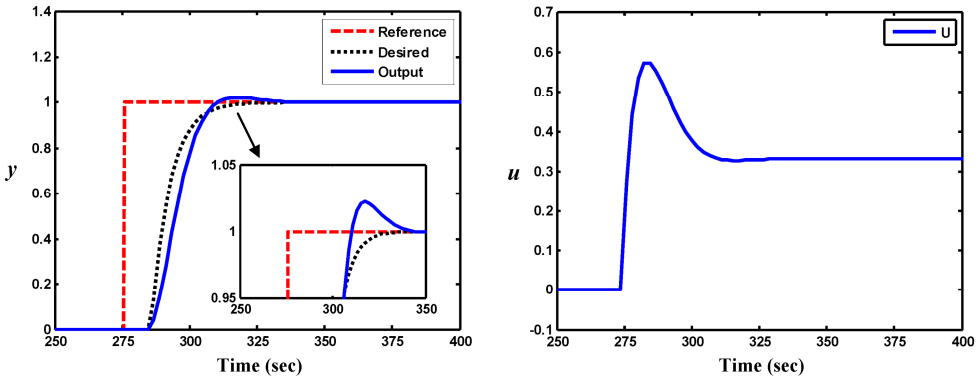

Figure 2. Closed loop response for $r = 3$.

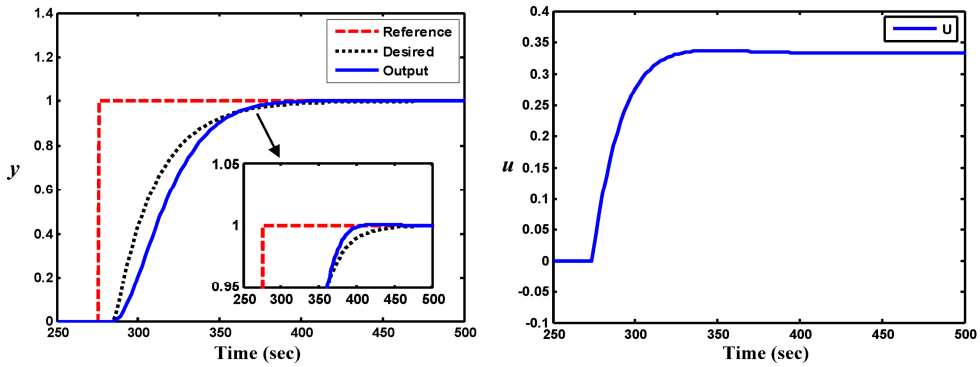

Figure 3. Closed loop response for $r = 0.8$.

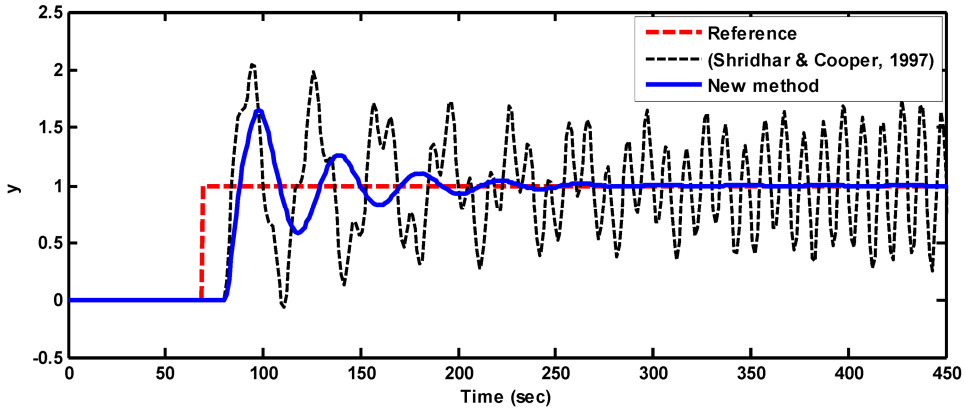

Figure 4. Closed loop response

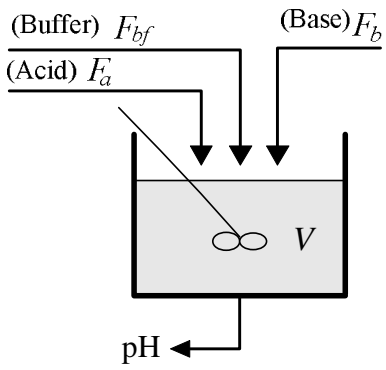

Figure 5. Schematic diagram of pH process.

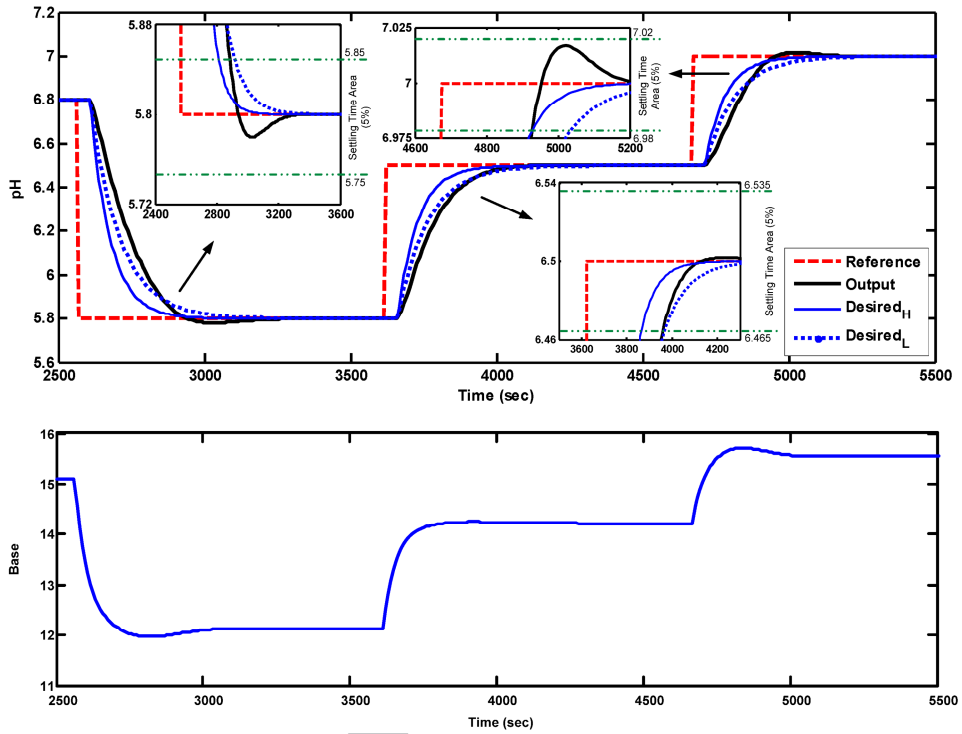

Figure 6. Robust Performance results for pH neutralization process

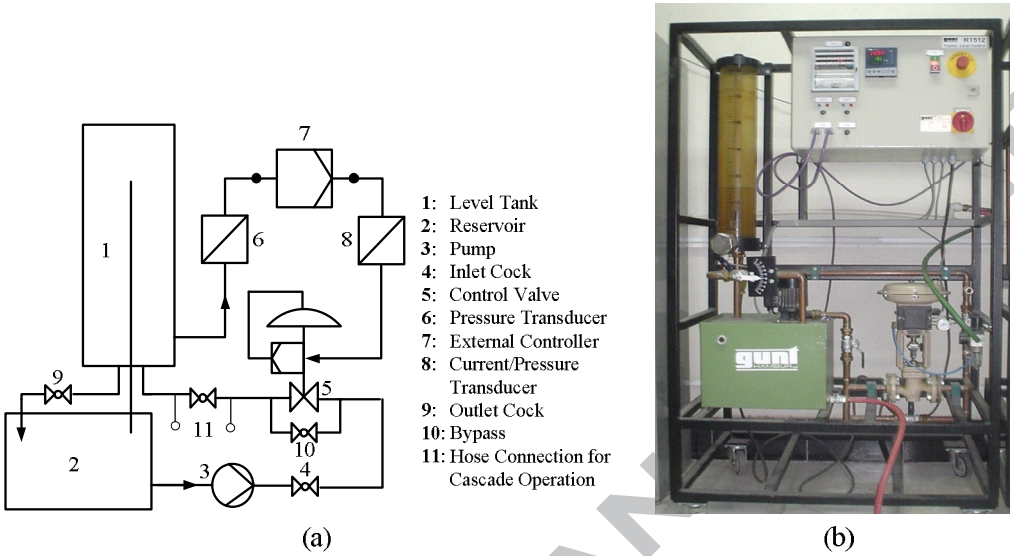

Figure 7. Lab-scale Level tank process, (a) P&ID diagram, (b) The real plant (RT512).

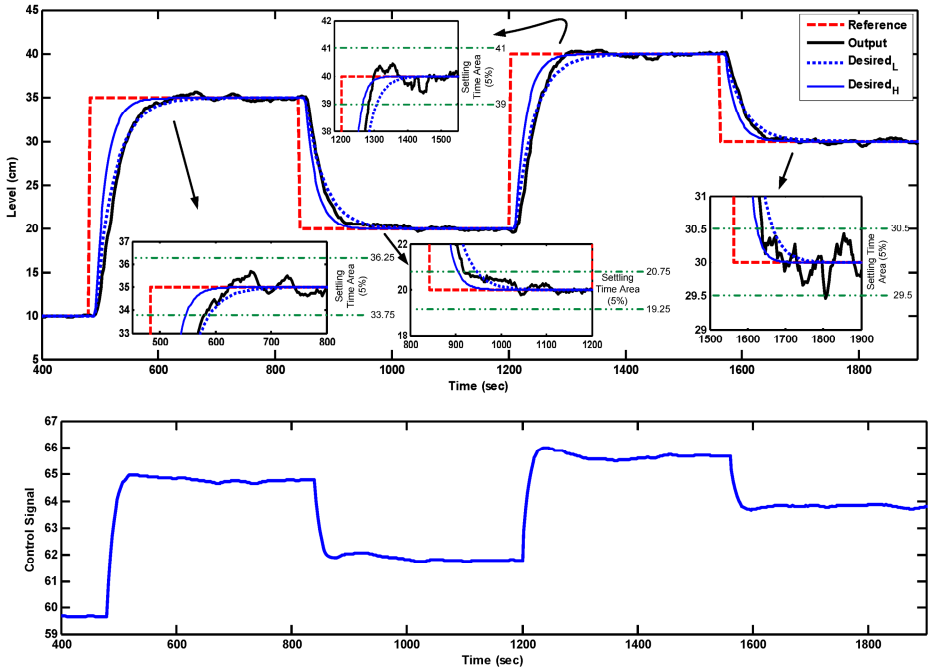

Figure 8. Robust performance results for level control system (first test).

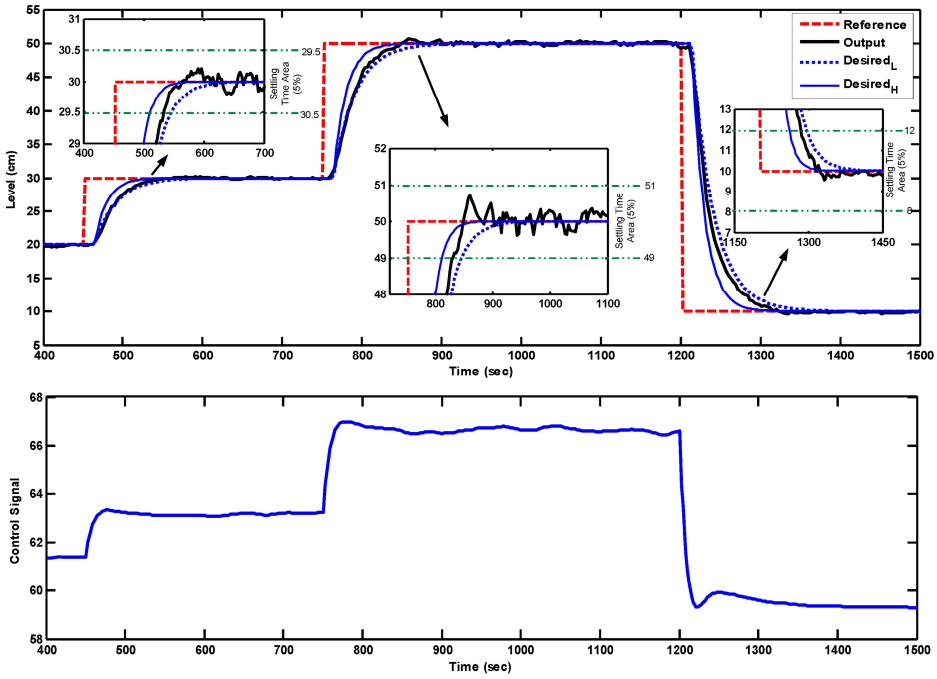

Figure 9. Robust performance results for level control system (second test).

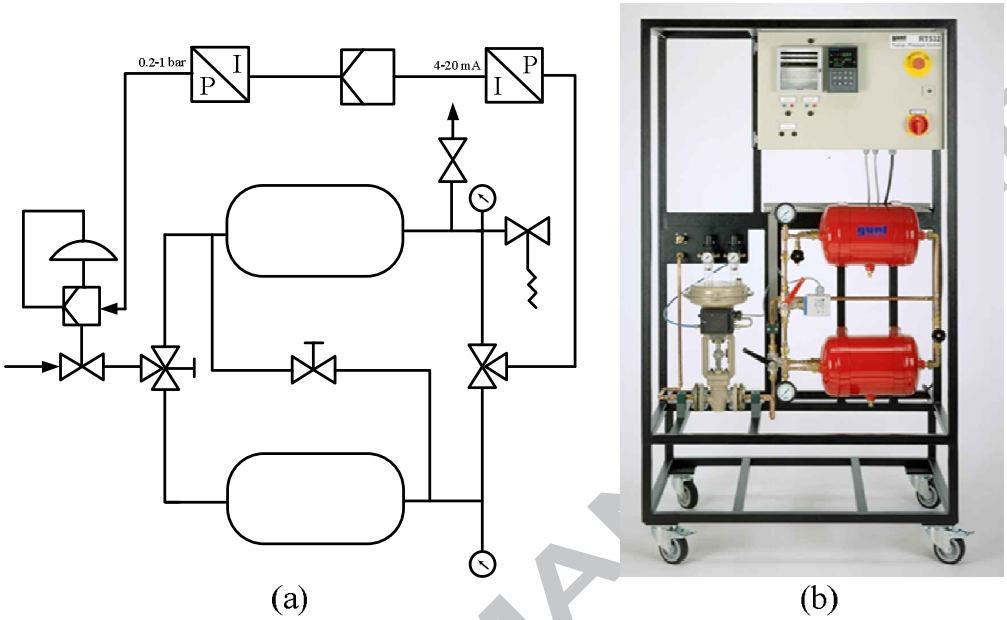

Figure 10. Lab-scale Pressure plant, (a) P&ID diagram, (b) The real plant (RT532).

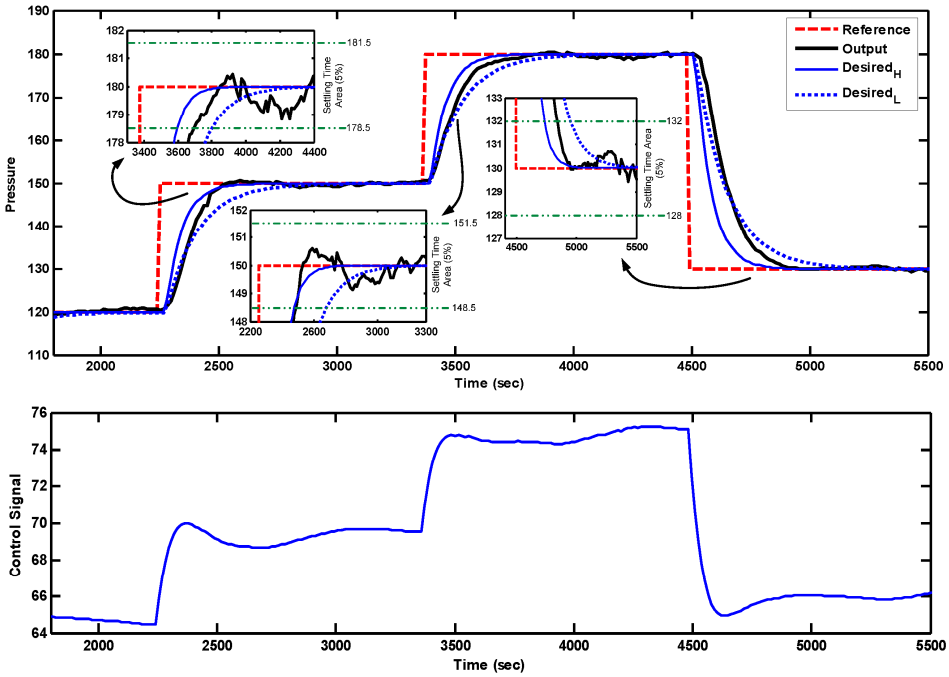

Figure 11. Robust performance results for pressure control system.

Table 1

Different DMC Tuning Equations.

Parameters	Wojsznis et al. [19]	Shridhar and Cooper [18]	Iglesias et al. [20]	Bagheri and Sedigh [21, 22]
T_s	-	$T_s \le \frac{\tau}{10}$ and $T_s \le \frac{\theta}{2}$	$T_s \le \frac{\tau}{10}$ and $T_s \le \frac{\theta}{2}$	$T_s \le \frac{\tau}{10}$ and $T_s \le \frac{\theta}{2}$
P and N	-	$P = N = \frac{5\tau}{T_s} + T_d$ $T_d = \frac{\theta}{T_s} + 1$	$P = N = \frac{5\tau}{T_s} + T_d$ $T_d = \frac{\theta}{T_s} + 1$	$P = \frac{5\tau}{T_s} + T_d$, $N = 2P$ $T_d = \frac{\theta}{T_s} + 1$
M	-	integer from 1 to 6	integer from 1 to 6	integer from 1 to 6
λ	$9\left(1 + \frac{6\theta}{P} + \frac{3k\theta}{P}\right)^2$	fk^2	$1.631k\left(\frac{\theta}{\tau}\right)^{0.4094}$	fk^2
f	-	$\begin{cases} 0 & M = 1 \\ \frac{M}{500}\left(\frac{3.5\tau}{T_s} - \frac{M-5}{2}\right) & M > 1 \end{cases}$	-	$0.84\left(\frac{\theta}{\tau} + 0.94\right)^{0.15} \Gamma^{0.9}$

Table 2

ANOVA Results for DMC Tuning.

Source	Sum Sq.	d.f.	Mean Sq.	F	Prob>F
θ / τ	7369.9	3	2456.6	3.85	0.0151
r	518614	16	32413.4	50.8	0
Error	30624.2	48	638		
Total	556608.1	67			